Fear of Mathematics

James J. Asher
tprworld@aol.com
www.tpr–world.com

"I would like to ask you a question that requires an honest answer." After a pause, I slowly scan the faces of thirty students newly enrolled in my statistics course at San Jose State, the oldest public university in California. I continue with, "How many people in this classroom feel some anxiety about taking a course in mathematics?" There is some hesitation, and then almost all the hands go up. I say, "Please keep your hand up and look around you. Notice that you are not alone."

"If you watch me carefully you will realize that all I am doing in each class meeting is lowering your anxiety because you already know every concept in this math class. You were born with these simple ideas already programmed in your DNA. My job is to relax you and convince you that what I am saying is true by releasing what you already know."

As part of my lower-their-anxiety strategy, I learn the name of every student in the very first meeting of the class. Are students really anxious? Well, I once asked a woman in her forties who was required to take statistics to complete a master's degree, "Will you please tell the class your name?" There was a long pause and then she said, "It was right on the tip of my tongue." After the class, she confided to me, "I am terrified of anything mathematical. I know I'm going to fail!"

"And I know you're not. I know you are going to complete this course with an A or a B."

"Impossible! How do you know that?"

"I know because I have worked with hundreds of students just like you. First of all, you are a mature women who is conscientious and able to follow directions. All I ask is that you follow me step-by-step and come to every meeting on time prepared to the best of your ability.

"You are going to be thrilled when you discover that statistics is nothing more than the novel arrangement of arithmetic that you learned in elementary school. I guarantee that you will be successful. I have not missed yet. Why would I not succeed with you?"

At the end of the course. the woman whispered to me, "I didn't know I was that good at mathematics! Now I want to enroll in the advanced statistics course."

Even the Chief of Police was anxious

San Jose, California has the honor of being the safest large city in the United States. Our new chief of police, a graduate of San Jose State, confided to a group of emeritus professors that he had no intention of making a career in law enforcement. As an English major, he wanted to be a writer. He has a passion for the printed word and little attraction to mathematical symbols. However, to finance his schooling, he became a reserve officer in the San Jose Police Department, discovered that he enjoyed working the streets helping people, and after a number of years, was promoted to the job of "top cop." Then he leaned forward and confessed to the group, "I want to tell you that I appreciate all my courses at San Jose State, but the one I found to be most valuable as Chief of Police will surprise you. For a guy who loves words and shied away from mathematics, I treasure my course in statistics. It is absolutely invaluable in making presentations to policy makers."

Fear of mathematics is a nationwide disability

There are two subjects in school that engender the most fear in students of all ages-- foreign languages and mathematics. Leslie Hart explains this phenomenon as the result of "brain antagonistic" instruction which refers to well-intentioned instructors playing to half the brain, usually the wrong half." More about this in a moment.

As to foreign languages, of all the students enrolled, studies estimate that about 96 percent will "give up" before achieving fluency. The FBI recently revealed after 9/11 that they did not have one special agent in the bureau that was fluent in Arabic. They depend upon translators. In a global economy, it is insulting to always expect foreigners to negotiate with us in English.

More than 30 states have eliminated the requirement of a foreign language for graduation from high school. Most parents want their children to acquire another language or two, but from their own disappointing experience in school, they perceive the effort as a "waste of time." Better to take something useful such as ball room dancing or small appliance repair.

As to mathematics, something is wrong because America spends more on remedial math than all other forms of math education put together. In a study a few years ago by the National Assessment of Education with a quarter million students, about half of our 17 year-olds could not answer simple math questions such as this:

What is 10 percent of 30? Is it...?

a. Equal to 30

b. More than 30.

c. Less than 30.

d. I don't know.

What is fear all about?

To better understand how fear works, I want to explore a fear that is at the top of the list for most people---fear of public speaking. It may surprise you that even the most famous actor in England, Sir Lawrence Olivier, was terrified to speak in public. Before every performance on stage, Sir Lawrence was so nervous, he would "throw up." On the surface it seems ridiculous that a world class actor with years of extraordinary success on stage and in films would have an "irrational" fear of public speaking. But it may not be so irrational if you take a look at what is happening in each hemisphere of the brain.

Sir Lawrence's left brain was saying to him, "Yes, you enjoyed rave reviews for every performance you have ever done, but this time will be different. Some of those strangers sitting out there in the dark will stand up and heckle you. They will see right through you and recognize you as a 'hack.' They will hoot and hiss you off stage. You are feeling sick. Better tell the stage manager that you are too ill to go on this evening."

That left brain can cause mischief

If you think that Sir Lawrence is a rare case, let's consider 63 year old super star, Plácido Domingo, "The King of Opera," who receives non-stop ovations that last an unbelievable hour and twenty minutes. Not only that, but he is fully booked for the next three years. Yet, he confided on the CBS program *60 Minutes* that he "has never been satisfied with any performance." How can this be? It doesn't make sense. He receives validation for his work that any performer in the world would "die for," and yet he is not satisfied.

The explanation again is in the left brain. Our left brain is dedicated to keeping us safe and sane. It accomplishes it's mission by telling us, "Better to be safe than sorry," "Look before you leap," "An ounce of prevention is worth a pound of cure," "Stay out of harm's way," "Mind your own business," and "Stick with the tried and the true."

The left brain has a kind of radar that scans for danger. It scans for imperfections. It scans for flaws. It evaluates information to determine what is true and what might be hazardous. In the case of the super-star, his left brain may be saying, "Oh, oh. You have to perform again. They will expect a stunning performance. You know you won't be able to match what you have done in the past. The audience will detect this and hoot you off stage. You can't keep on being a hit. It is not humanly possible to be a smash every time. Look at what the New York Times said about you: 'Domingo has reached his peak and cannot continue much longer.' Yes, that article appeared 23 years ago, but now—now, they may be right."

The right hemisphere of the brain

To better understand what is happening on the left side of the brain, let's compare it with the right hemisphere. The right brain is a mirror-image of the left brain. The left does the talking and processes symbolic messages such as words in print. Sigmund Freud thought that part of the brain that we now identify as the right brain is subconscious or unconscious. That is an illusion, in my opinion, coming from the fact that the right brain cannot talk. The entire brain is conscious, but we seem to be aware only of messages we hear "loud and clear" from the talking left hemisphere.

Because the right brain cannot talk, we have the misconception that it is turned off or "unconscious." *It is conscious all the time,* but it does not have a voice box to communicate so it tries to send us messages in other ways such as drawing, acting, body movements (watch people's hands when they talk), stories, and dreams. We believe that whispering and singing (for reasons we still do not completely understand) happen on the right side of the brain.

While the left brain is critical and logical, the right brain does not evaluate and therefore, does not filter information. We get a stream of raw data from the right brain without any editorial comment or censorship. The right brain seems to follow directions literally in an attempt to find solutions or ways to reach our goals. There is some truth to the adage, "Be careful what you wish for. You may get your wish."

Schizophrenia may be what happens when the left brain is turned off (for reasons still not known) so that there is no "reality check" on the stream of ideas spilling out from the right brain. Think how schizophrenic we would appear to others if we expressed every thought that came into our heads. For example, men and women carefully filter out thoughts about each other. What we say has been "homogenized" by the left brain to be "civilized," "acceptable," and "pleasing" to the listener.

How the right brain works

The code the right brain uses to process information is still a mystery that perhaps will be solved in this century. I believe the solution will be more exciting than any discovery in the physical sciences because it has profound significance for the optimal use of the brain to solve problems. The survival of our species may depend upon understanding the riddle of how the right brain works.

Sigmund Freud was on the right track in his attempt to recover and analyze a patient's dreams to discover clues for resolving personal problems such as "irrational" fears and anxieties. Most psychotherapies including Freud's psychoanalysis, operate on the right side of the brain with the formula: "Find it. Face it. Erase it." To give you an inkling of how amazing the right brain is, I would like to share a personal story: One day my wife remarked that she wanted to lose weight, and I responded that I didn't think it was necessary. The next day, my wife said, "I had the strangest dream. I dreamed that the dentist was filling my tooth with a miniature doorbell. What does it mean?"

I said, "Sigmund Freud would 'knock this one out of the ball park.' If you have a tiny doorbell in your tooth, then every time you chew, the doorbell will ring. This means you will not chew as often and therefore, you will not consume as much food resulting in losing weight." It is a silly solution but a solution, nonetheless. Remember, the right brain does not evaluate. Anything is possible. Unless we instruct the right brain to abort with, "I give up. I guess there is no answer," the right brain will continue to search for solutions until we exclaim, "Eureka! That's it! I just got a great idea!"

As to dreams, when we awaken, the left brain examines the dream and concludes, "This is unbelievable. It does not make sense." And the dream immediately begins to break up and disappear from awareness.

Now, let's apply all this to the fear of mathematics

I once remarked to an audience that nobody in the auditorium has ever had a course in mathematics. "What do you mean?" an indignant man in the front row told me in confrontational tones. "We have all had many courses in mathematics."

No, I don't think so. What we have experienced was not mathematics but what I call, "shadow mathematics" because all that stressful manipulation of numbers was like galley slaves pulling back and forth endlessly on oars to create in the left brain of students one false belief on top of another. For example, there is the crazy belief that there is only one way to get the answer in mathematics and all problems can be solved with a step-by-step method. And there is the bizarre belief that math is mostly memorization and it is hard—too hard for most people to learn. Now for the *coup de grace:* There is a math gene—some have and others do not. Hence, only geniuses are capable of creating or understanding formulas and equations. Of course the left brain scans all those beliefs and to protect the student from failure, advises: "Run fast; run far! This is not your cup of tea."

Keepers of the secret

Professional mathematicians have some secrets which are not revealed until one becomes an advanced graduate student. For example, explanations in textbooks are imperfect. If you don't get it, the chances are the instructor doesn't get it either.

Another secret: Try an analogy

Analogies play to the right brain and are exciting because they enable understanding in the very first exposure of any concept in any field. A classic example is this question: How long have dinosaurs been on earth? The left brain answer is: 100 million years. The student can memorize the answer and get a perfect score on a test with absolutely no understanding.

To actually communicate to students how long dinosaurs have been on earth, try this analogy that plays to the right brain: If the age of the earth is 24 hours, dinosaurs were here for one or two hours and we have been on earth one or two minutes. If an abstraction can be converted into an analogy, any student can "catch the instructional ball when the instructor pitches it." Most students can get it in the first exposure without memorization and without stress. In mathematics, analogies are rare in print and rare in the classroom.

Here is another example: A teacher asked an 8th grade student, "Luke, tell the class a definition of infinity." The boy said, "Infinity is a box of Cream of Wheat." The instructor was expecting a left brain textbook answer and replied, "Luke, let's get serious." But Luke was serious because a box of Cream of Wheat illustrates the exact nature of infinity. The reason: On the box is a picture of a chef in white outfit and chef's hat holding a box of Cream of Wheat. The box in his hand has a miniature picture of a chef holding a box of Cream of Wheat and so on into eternity.

The absence of analogies means that students are confined to using half the brain, and usually it is the wrong half. By imprisoning students in their left brain, we mystify them with one abstraction after another. Since there is almost no brainswitching from one side of the brain to the other, the result is "brain antagonistic" instruction. All of this produces a perfectly understandable avoidance reaction to math. The left brain urges, "Run fast! Run far!" Some students with "academic aptitude" can brainswitch on their own without any assistance from the instructor, but most people need help. This is the art of being a successful instructor. This is the instructor we remember even if we live to be 90 years old.

60 Minutes ran a piece about a Harvard professor who was concerned about the prestigious school always selecting left brain students who get into Harvard because they are all "A" students, meaning they are Mozart's of textbook learning. To encourage right brain stimulation, he created a course I call, The Harvard Walk. It is so simple, it drives students crazy. He takes them on a walk for an hour or so and points out small details on manhole covers or doorbells or patterns on glass. He asks them to pay attention to sights and sounds and smells instead of always being a prisoner inside a world of words and mathematical symbols.

An illusion about mathematicians

When math is presented in the classroom or in a textbook, it is often a formal, precise, and disciplined step by step progression to a logical conclusion. By contrast, spoken communication between mathematicians is often "fuzzy, sloppy, and non-linear" according to Loats and Amdahl who wrote a book with the wonderfully giddy title, "Algebra Unplugged."

Students watching mathematicians perform in the classroom come away with the illusion that mathematicians are precise, formal, and accurate dispensers of absolute truth. The reality is that professional mathematicians are imprecise, indefinite game players who doodle with numbers and relish exploring options. It was the Alfred Lord Whitehead, who wrote prize-winning books about mathematics and remarked that "Mathematics is a subject in which we do not know what we are talking about and whether what we are saying is true."

The Ultimate Illusion: Those maddening word problems

Those word problems pretend to be actual real life problems that one could encounter in everyday living. The truth is that are completely artificial, convoluted nonsense compounded to give students practice in applying algebraic code. The reason they are artificial is the difficulty finding actual problems in everyday living that require algebra. I have a standing challenge to anyone in the world to e-mail me with an actual problem that requires algebra to solve. The few that I have received and submitted to professional mathematicians for evaluation turn out to be simple problems in arithmetic.

A classic example of a nonsensical word problem

The ages of Ellen and her sister add up to 18. Ellen is twice the age of her sister. What is the age of each girl?

First, the writer knew the answer before writing down the question. You have to know the ages of Ellen and her sister before you can conclude that Ellen is twice the age of her sister. If you know the answer in advance, why ask the question? Second, nobody cares about the ages of Ellen or her sister. So, here we have an artificial "problem" that is not a problem at all, but a puzzle.

Puzzles can be fun to play with on a rainy afternoon, but only if we know they are puzzles. If they are presented as something the student may very well encounter in later life, the student's brain, below the radar of consciousness, will recognize this as pure "nonsense." The student is not aware exactly what is happening, but has a vague feeling of uneasiness—something is not quite right.

This conundrum about the ages of little girls was concocted "out of thin air" to give the student practice in applying algebra step-by-step like this:

What do we know?

1. Ellen is twice as old as her sister.
2. So, if her sister's age is (the mysterious) X, then Ellen is 2X.
3. We are ready to write (the mysterious) equation:
   Ellen's age + her sister's age = 18
   or
   2X + X = 18
4. The objective is to get X on one side and everything else on the other side of the equation.
5. So, 3X = 18
   X = 18/3 or 6
6. Voila! Ellen's sister is 6 years old.
   Hence, Ellen must be 2 times 6 or 12 years old.

Notice that this conundrum can only be solved one way because it was constructed to give a single answer. Therefore, the student gets the false impression that there is only one answer to any math "problem," and furthermore, there is a mechanical step-by-step procedure for arriving at that answer.

The truth is, mathematics is one of the most creative ventures in the world, with discoveries made by doodling, scribbling, erasing, crossing out, and drawing pictures. Mathematicians explore mysteries that have no ready-made answers such as:

The fundamental theorem of arithmetic

(A theorem is an unproven belief thought to be true)

All numbers other than primes are composed of prime numbers.

For example, 4 = 2 times 2… and 2 is a prime number.
6 = 2 times 3… and both 2 and 3 are prime numbers.

If the theorem is true (and I personally question it), then primes are the basic building blocks of all mathematics. Primes are the DNA of mathematics. Something that fundamental to all of mathematics, has to—just has to have a pattern. The mystery is that no one yet has discovered the hidden pattern such as a formula for predicting all primes or even a formula for predicting some primes.

Why must there be a pattern?

Albert Einstein gives this explanation: "God (Nature or whatever you wish to call the intelligence of the universe) is subtle, but not malicious." Cause-effect patterns have to exist to explain everything because if everything in the universe is the product of randomness, science would be impossible. Further, Einstein implied that God is not cruel. As we get close to revealing a hidden pattern, God will not change the rules to "trip us up." "God is subtle, but not malicious."

If the universe is random, science is impossible.

Let's try an analogy that Dr. Einstein would have liked. If every person living or who has ever lived in the world were to hold a deck of cards, each holding a different arrangement of the cards, there would be no two people holding the same "hand," and furthermore, there would be many patterns still available for people yet unborn. This is expressed as 52!, which represents every possible arrangement of 52 cards, arrangements which number into trillions.

If everything is random, then it is futile to pursue any mystery in medicine, physics, biology or any other field. To try and locate an item hidden in a territory with trillions of addresses in random order makes looking for a "needle in a haystack" child's play. Where does one look when there are trillions and trillions and trillions of options? One does not live long enough to explore all those options.

Back to the word problems:

Here is what is happening in the student's brain

Below the level of awareness, the student's brain makes a high- speed microsecond evaluation of the word problem and concludes that it is "nonsense." It does not make sense. Then the brain makes a secondary evaluation: "But the instructor and other students seem comfortable. They seem convinced that the problem is relevant and worthwhile. It must be something wrong with me. I don't understand. I guess I'm no good at math." Leslie Hart calls this "brain antagonistic" instruction.

How to get "brain compatible" instruction

The situation can be transformed into "brain compatible" instruction if the instructor introduces the word problem with this disclaimer, "The next problem will make no sense to you. It is frankly nonsense, but it will give you a chance to practice applying some basic algebraic principles. So, please do not take the problem seriously. Think of it as a fun puzzle rather than an important problem that will change the world."

That is exactly the instructional strategy that world famous Nobel-prize winner, Dr. Richard P. Feynman used with his students in physics. Dr. Feynman would introduce the topic of light like this, "The next thing I am going to say will make absolutely no sense to you. It makes no sense to professional physicists, but here is what we have discovered about light that is so puzzling and fascinating at the same time..."

Then he tells his students about particles of light called photons. When you flash a light through a pane of glass, perhaps 8 in 10 photons will decide to travel through the glass and two will decide not to and bounce back off the glass. Now, here is the mystery that drives physicists crazy: The next time you flash the light, 6 photons travel through the glass and four refract or bounce back from the glass.

Why is it that each time we flash a light at the glass, some photons decide to travel through the glass and others decide not to make the trip. It looks like a random event, but is it? Is there a hidden pattern we have not yet discovered? We are searching for the pattern that will predict which photons travel through the glass each time we flash the light. And why do some light particles decide not to make the journey through the glass?

Another Illusion

"There is only one answer to mathematical problems."

There is a belief in science and mathematics that nature (God, if it pleases you) prefers simple principles that are symmetrical. As one mathematician expressed it, "If your mathematical theory is so complex that you cannot explain it to a child, it is probably false. Keep working on it!"

Standard arithmetic that we all learned in school makes this assertion: ***Multiplication is simply repeated arithmetic.*** For example, if we add 2 + 2, we get + 4 and if we multiply 2 times 2, we get + 4. So students become convinced that multiplication is simply repeated addition. Not only is multiplication supposed to be repeated addition, but notice the elegant symmetry between addition and multiplication. It seems as if we can go from addition to multiplication or from multiplication to addition.

The myth that multiplication is repeated addition

Let's play with this and see what happens:

Addition		Multiplication
2 + 2 = + 4	and	2 times 2 = + 4

We go from addition to multiplication
and get the same answer. So far, so good.

Let's doodle with negative numbers and see what happens:

Addition		Multiplication
(- 2) + (-2) = - 4	but	(-2) times (-2) = + 4.

Wait a minute!
We now get a different answer
when we go from addition to multiplication.
What happened to symmetry?
Why isn't (-2) times (-2) = - 4?

Let's mess around with it when both positive and negative numbers are mixed:

Addition	Multiplication
2 + (- 2) = 0	(2) times (- 2) = - 4

Whoa! This does not make sense!
Clearly, multiplication is not simply repeated addition.

Let's tinker once more with negative and positive numbers:

Addition	Multiplication
(- 2) + 2 = 0	(- 2) times 2 = - 4

Again, this really does not make sense
if multiplication is simply repeated addition.
What is the explanation?

Conclusion:

Standard arithmetic gives us four patterns that do not make sense

Pattern 1 is symmetrical.
2 + 2 = + 4
and
2 times 2 = + 4

Pattern 2 is asymmetrical.
(-2) + (-2) = - 4
and
(-2) times (-2) = + 4

Pattern 3 is asymmetrical.
(-2) + (+2) = 0
and
(-2) times (+2) = - 4

Pattern 4 is asymmetrical.
(+2) + (-2) = 0
and
(+2) times (-2) = - 4

Hidden Symmetry for Addition and Multiplication

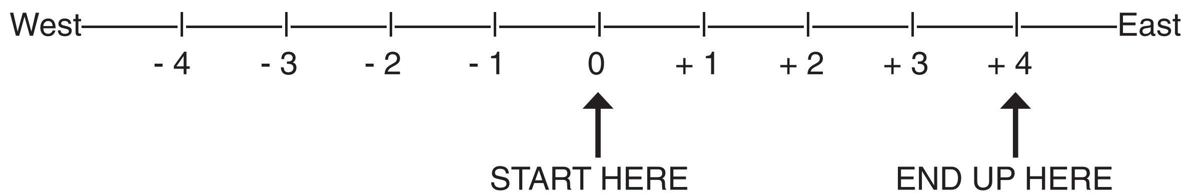
I challenge the assertion in standard school arithmetic that "multiplication is simply repeated addition." I believe that this premise only applies to positive numbers.

I believe there is an elegant symmetry for addition and for multiplication if we transform equations (which are processed in the left brain) into pictures which are processed in the right brain like this:

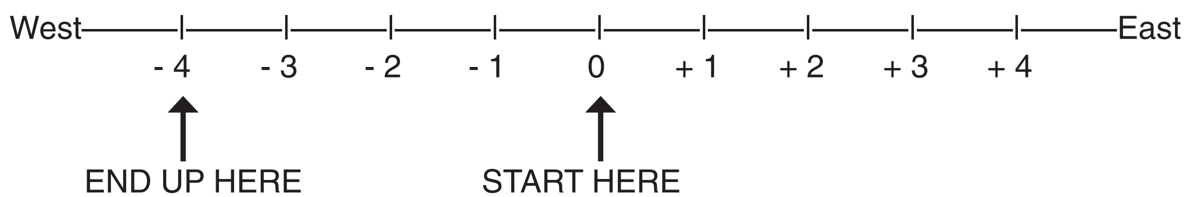
There is a marvelous symmetry for addition

Addition has marvelous symmetry if you picture addition for positive and negative numbers as walking either East or West on the number line:

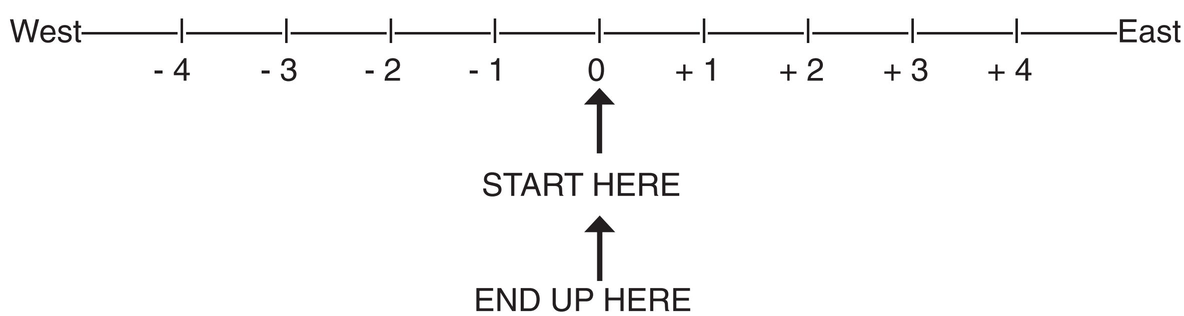
Example A: 2 + 2 tells you to walk two steps East and another two steps East.



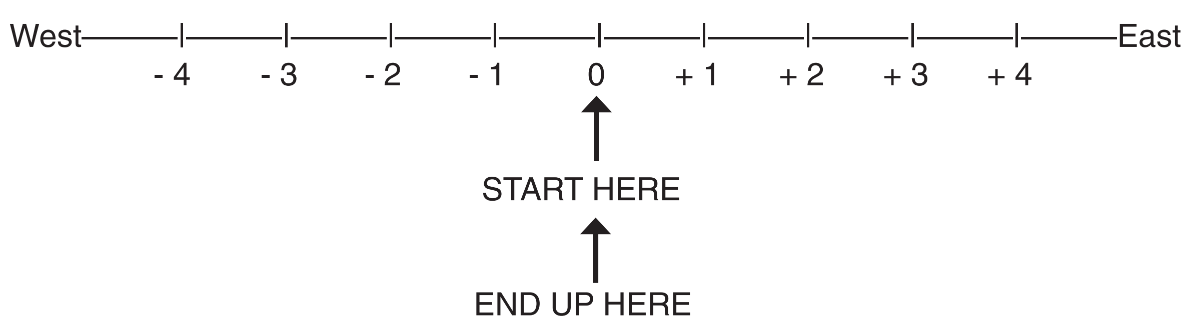
Example B: (- 2) + (- 2) tells you to walk two steps West and another two steps West.



Example C: (+ 2) + (- 2) tells you to walk two steps East, turn and walk two steps West. You end up where you started.

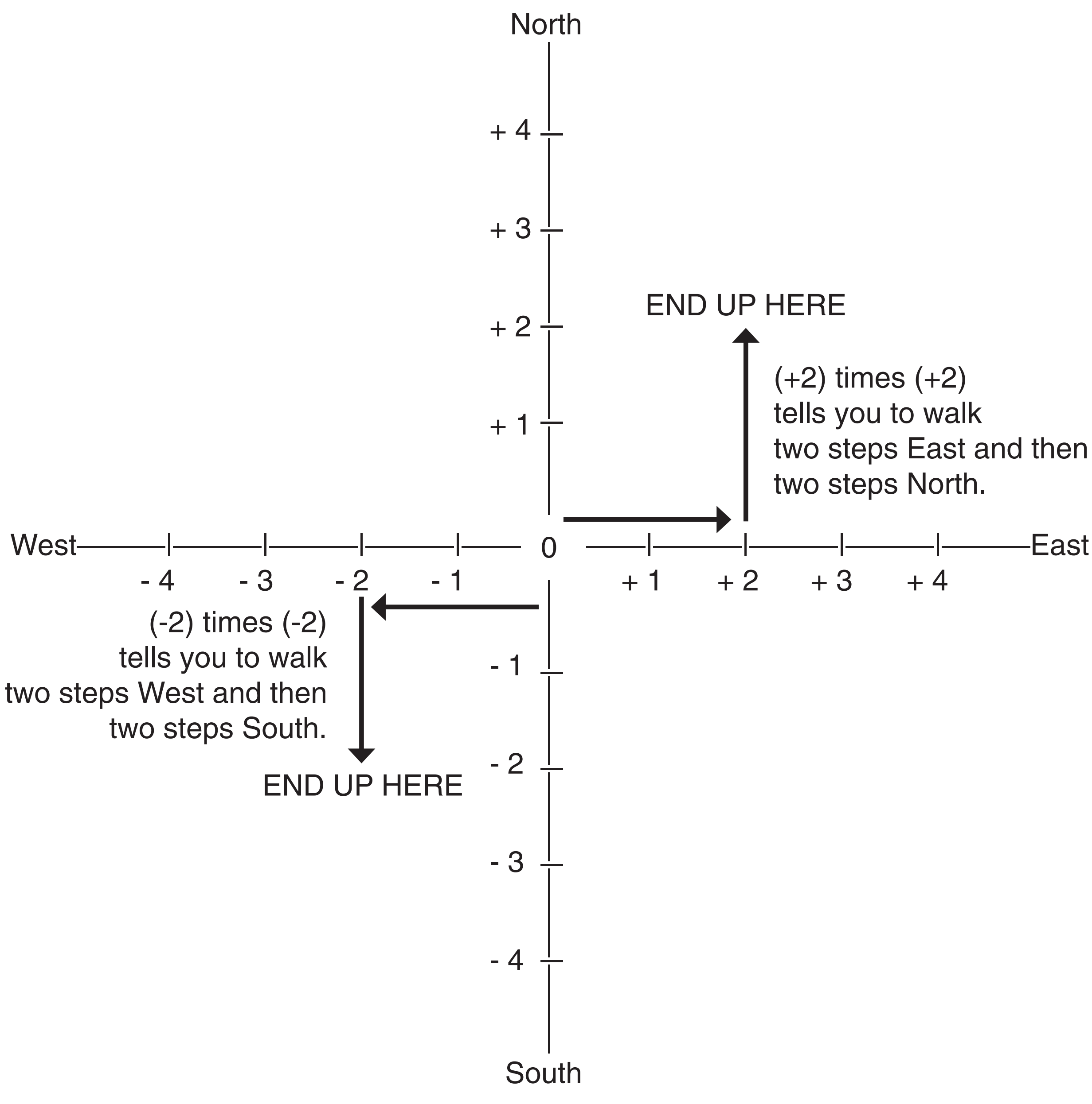


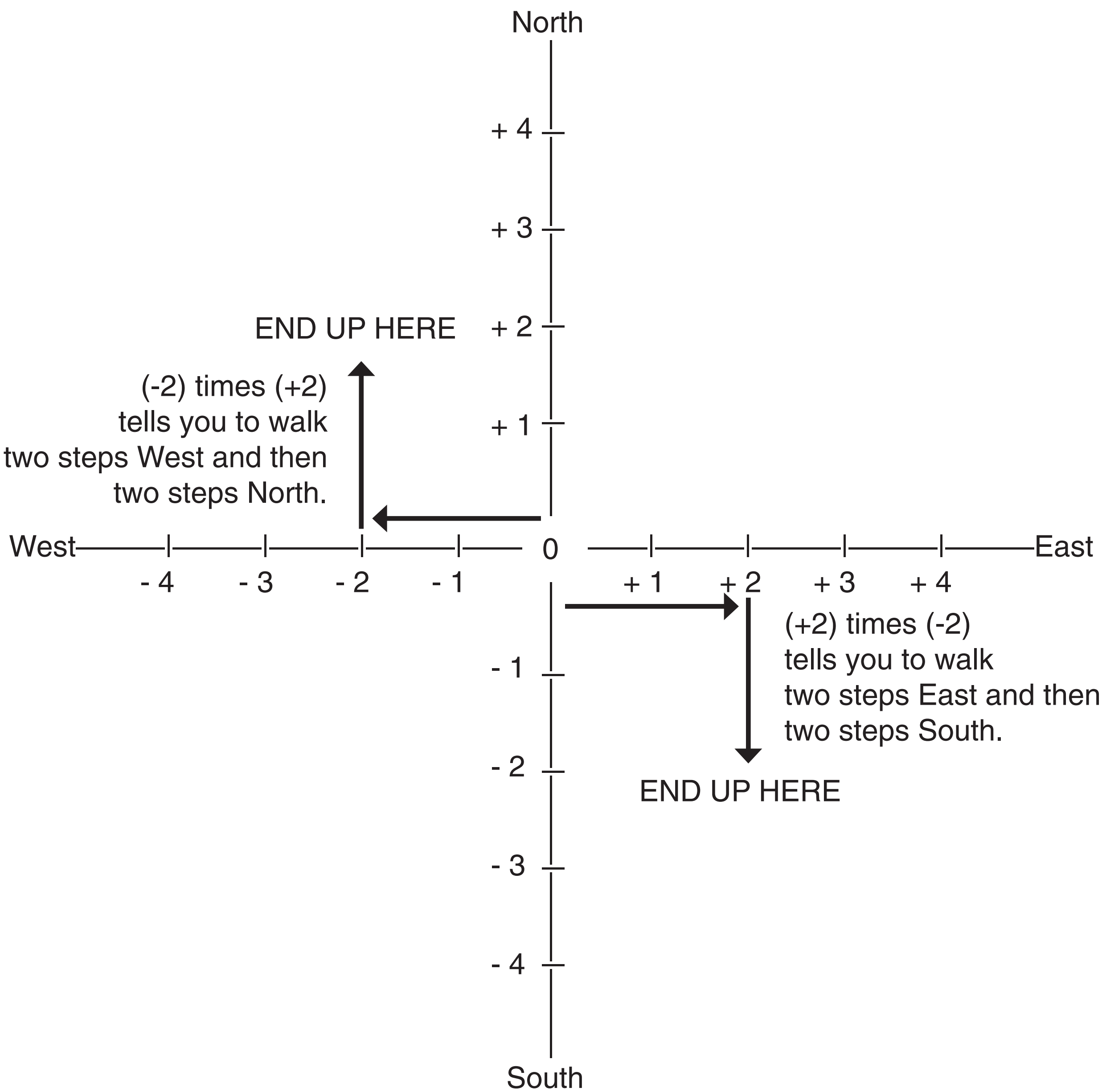
Example D: (- 2) + (+ 2) tells you to walk two steps West, turn and walk two steps East. You end up where you started.



There is also a marvelous symmetry for multiplication

Multiplication also has marvelous symmetry if you picture your walk in the direction of North and South as well as East and West like this:





Notice the elegant symmetry in which the four relationships fit together perfectly.

Conclusion: The model of arithmetic that we all experienced in elementary school has many flaws when students are encouraged to examine it, to play with it, to question it.

The alternative model I have presented is not *the answer* but another possible interpretation which has many more attractive features compared with the standard school model. Notice that arithmetic is not a closed book. It is wide open for exploration. The most exciting discoveries have yet to be made, perhaps by students now in elementary school.

The next myth:
Only geniuses can understand formulas and equations

The story of Sir Isaac Newton, perhaps the greatest mathematician England ever produced

I mentioned the widely held belief that anyone who understands mathematics must be a genius. It is amazing then that Sir Isaac Newton, perhaps the greatest mathematician that England ever produced, was next-to-the-lowest ranking student in Grantham's Free Grammar School of King Edward VI. But, like Albert Einstein and Thomas Edison, he was unusually inquisitive. When Einstein was once asked what were his talents, he responded, "I have no particular talent except I am extremely inquisitive." Newton, Einstein and Edison did not shine as young students, and in particular, they did not shine in mathematics. When Newton entered Cambridge at the age of 19, he had very little preparation in mathematics beyond simple arithmetic.

Newton was fascinated with the movement of the planets. To understand that motion, he realized that algebra and trigonometry were important. So, from books, he taught himself those mathematical skills and began a quest to discover the velocity of planetary movement at any given instant. In the 16th century there was nothing to guide him. There were no ready-made formulas or equations. So, most scientists simply threw up their hands and said, "I give up! It can't be done! It is impossible!"

But Newton somehow tinkered and doodled and scribbled in scores of notebooks until he finally discovered how to do the "impossible." He discovered how to measure an "instant," which is the jewel of mathematics we now call "calculus."

As Newton began to tinker with possible ways to measure an "instant," he was blocked by the concept of zero. Here is how it works: From the stunning demonstrations of Galileo, he knew there was a way to measure the distance of something such as a stone falling from a height. Galileo showed that if you know the time it takes for the stone to hit the ground, you will automatically know the distance the stone traveled if you simply square the time.

Here is another mystery for a student in elementary school to play with. Why does time squared tell us the distance an object falls? Why not time cubed? Or time taken to the 4th power? Why did nature prefer time squared?

Galileo's law is… time squared = distance

In algebraic code, Galileo's law is $t^2 = d$

For example:
If the time is 2 seconds, then the stone traveled 2^2 or 4 feet.

If the time is 3 seconds. then the stone traveled 3^2 or 9 feet.

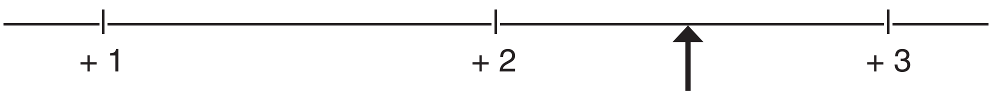
If the time is 4 seconds. then the stone traveled 4^2 or 16 feet.

But how far does the stone travel in an "instant"?

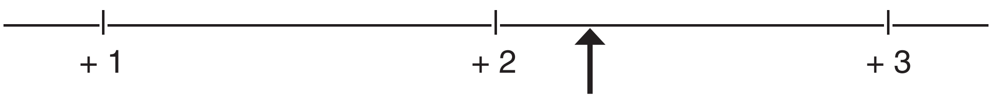
How can we represent a time interval of an instant? Since an instant is so close to zero, let's try a time interval of zero. No, that will not work because zero squared is zero which means that the stone did not move. An instant is close to zero but it is not zero. Zero is nothing and an instant is something, but what is it?.

Here is the secret that Newton discovered

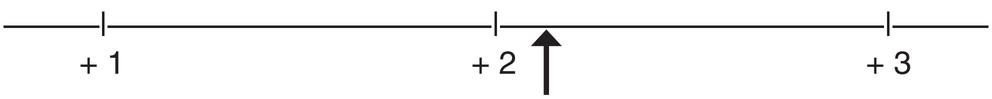
As an illustration, starting at 3 seconds, how close can we get to 2 seconds without ever reaching 2 seconds? Try 1/2 step back from 3 seconds in the direction of 2 seconds.



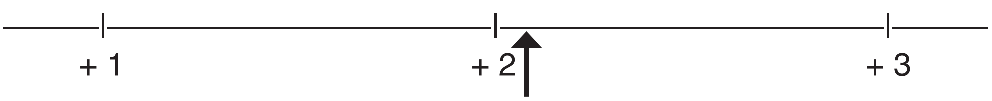
Next, try 1/2 step back from there and you are 1/4 away from 2.



Try 1/2 step back again and you are 1/8 away from 2



Try 1/2 step back again and you are 1/16 away from 2.



Notice with each 1/2 step back you are getting closer to 2.

Will you ever reach 2? The answer is never. There will always be some speck of time left, no matter how many 1/2 steps you take. That speck of time is often called "delta t", and is represented by the symbol **Δt,** or simply **dt.**

What is an "Instant"?

2 seconds plus *a minute increment* <u>minus</u> the original 2 seconds will *almost* tell us how far the stone travels in an "INSTANT." Let's try it with algebra:

$(t + dt)^2 - t^2$ = distance *almost* traveled in an INSTANT

↑ *speck of time*

Here's the secret of calculus

The fly in the mathematical ointment is this: The minute increment of **dt** is not a fixed amount, but it is in *continual motion,* getting smaller and smaller forever as it approaches the value of **t**, which is the famous concept of a "limit." "**t**" is the limit for **dt**, but **dt** will *never reach* **t**. "**dt**" will get close to **t** but never, never ever reach it.

So, **dt** is a value getting smaller and smaller without ever stopping. Hence, we need some average for the deceasing increment of **dt**. A way to get the average is to take $(t + dt)^2 - t^2$ and divide it by **dt**.

The algebra looks like this:

$$(t + dt)^2 - t^2 =$$
$$t^2 + 2tdt + (dt)^2 - t^2 =$$
$$2tdt + dt^2 / dt =$$
$$2t + dt$$

dt is a small number getting smaller as it recedes into eternity. Newton and Gottfried Liebnitz in Germany said, "Since it is smaller than any number and getting smaller and almost vanishing, we can remove it and only accept **2t** as the distance the stone travels in an "INSTANT."

With each half step back you are closer and closer but you will continue forever without reaching 2, in our example. So, an INSTANT (often represented as **dt**) is a continuous 1/2 step back in the direction of 2. This is a microscopic speck of time so small that we will never know exactly how small, but **dt** is so infinitely tiny as to be almost non-existant and hence, Newton reasoned, we can safely *erase* it from our computation. Therefore, the distance of **2t + dt** becomes **2t**.

Some practical examples

In 10 seconds, an object travels, according to Galileo's Law, t^2 or 100 feet and in the instant following 10 seconds, the object travels, according to Newton's calculus, 2t or 20 feet.

In 30 seconds, an object travels, according to Galileo's Law, t^2 or 900 feet and in the instant following 30 seconds, the object travels, according to Newton's calculus, 2t or 60 feet.

In 60 seconds, an object travels t^2 or 3,600 feet and in the instant following 60 seconds, the object travels 2t or 120 feet.

This was a stunning leap of logic that was not without its critics. When Newton shared this concept with the eminent members of the Royal Academy of Scientists, his presentation was received with "stony silence." The concept was so deviant from any know mathematical model, the audience thought perhaps the great Newton had become "insane." Newton was so hurt, he hid his notes for twenty years, and only when friends told him that Liebnitz in Germany was publishing something similar, did he decided to present his ideas in print.

Even then, there was non-stop criticism as when Bishop Berkeley said it was folly to throw away a remainder, no matter how small. He called Newton's increments, "Ghost numbers." Liebnitz himself called the increments, "fictitious." Berekely admitted, however, that even though the concept seems to defy the axiom that, "In mathematics not even the smallest errors are ignored," the answer for instantaneous velocity seems to be correct.

Conclusion

The picture students have of mathematics and mathematicians is most certainly fictitious. Math textbooks and classes represent a “homogenized” view of mathematics that in no way showcases the exciting stories of how discoveries are made. The stories are the romance of mathematics which should come first to inspire students so they are ready and eager to continue exploring the mysteries of mathematics.

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