The Myth of Algebra

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founded in 1857 as the oldest public university in California.
SJSU has 30,000 students and a faculty of 1,800 Ph.Ds.

Fifty years from now, educators may conclude that algebra for every boy and girl may very well qualify as “cruel and unusual punishment” for our children. Granted, algebra is a powerful tool for a handful of occupations such as engineering, science, and mathematics. But, to mandate that every boy and girl be proficient in algebra with the assumption that it improves “thinking” and “problem solving” is a myth. We made the same assumption about Latin 100 years ago.

The evidence is clear-cut that neither Latin nor algebra sharpens intellectual abilities. Rather than being an elixir that improves the well-being of students, algebra does harm because one of every two students experiences “failure,” and then they jump to the false and debilitating conclusion, “I guess I am no good at mathematics.” Algebra is one reason we spend more money on remedial math than all other forms of math education combined.

I think that Dr. Jim Loats and Kenn Amdahl put their finger on the problem in their fine book, Algebra Unplugged. They said, “There is something about the way math classes and math books are organized that does not make sense to most students.” In this article I want to reveal what there is about one component of algebra, those “word problems,” that does not make sense to most students.

Nobody cares about word problems—not even the writers of algebra textbooks The reason word problems in those textbooks are so nonsensical is that it is impossible to find meaningful problems in the real life of ordinary people. Since none exist, writers of math textbooks must invent convoluted word problems that are of no interest to anyone, including those who wrote the textbook. Let’s start with Ellen and her sister.

Ellen and her sister....

Ellen is 7 years older than her sister, and the sum of their ages is 21 years. How old is each? First, this is a puzzle and not a problem because no one cares one way or the other about Ellen or her sister. Secondly, not only is the answer already known, but the answer came before the question. Unless you already know the ages of both Ellen and her sister, how can you conclude (a) that Ellen is 7 years older than her sister, and (b) the sum of their ages is 21 years? The ages of Ellen and her sister had to be known in advance. So why ask the question?

Writers of algebra textbooks: Let's see how their minds work

They start with an authentic question and convolute it to produce a nonsensical puzzle that requires some algebraic code to solve. Since the word problem will appear in a respected textbook and it is in print, the student wonders why the "problem" does not make sense and concludes that, "I don't get it! I guess I'm no good in algebra! No, make it—I'm no good at mathematics!"

Here is an authentic question about Ellen and her sister

Ellen is 14 years old and her sister is 12 years old. Movie tickets are $6 for general admission and 1/2 price for children age 12 or younger. How much money should mother give the girls to cover both movie tickets? Ellen's ticket is $6 and Ellen's sister will be 1/2 of $6 or $3. So, the cost of both tickets is $6 + $3 = $9. So far, so good. You probably answered that question without pencil and paper. Now, convolute that simple problem in arithmetic to create a synthetic puzzle that requires some algebraic code to arrive at an answer. Convoluting the simple to create the complex calls for maximum creativity from the writers.

The convoluted version

The cost of movie tickets for Ellen and her sister is $9. Since Ellen's sister is 12 years old, she pays only 1/2 price of Ellen's ticket because Ellen is 14 years old. How much is the movie ticket for each girl? Now we already know the answer: Ellen's ticket is $6 and her sister's ticket is $3. But how do the writers apply some algebraic code to find the $6 and the $3? Let's play with it.

Cost of Tickets for Ellen and Her Sister	=	**$9**
Cost of Ellen's Ticket	=	**X**
Cost of Sister's Ticket	=	1/2 of X or **.5X**

Now we can write an equation: $**9 = X + .5X**

Remember that an equation is like flying an airplane. As long as the wings are level, we are on a safe flight path. You know the wings are level if the equation has an = sign. Once you turn the wheel, the plane is slanting with one wing higher than the other. This happens in an equation when you add, subtract, multiply or divide something from one side of the equation. To get the plane back on a safe flight path, turn the wheel in the opposite direction by quickly performing the same arithmetic operation on the opposite side of the equation. Ahh. Once again, the sign on the dash of the cockpit reads = , which signals that the wings are level and we are flying in a safe flight path. The entire mission of any equation is to fly the plane with the wings level.

Let's translate the equation into English

The plane is flying with the wings level. The tickets for Ellen and her sister cost **$9**. The **$9** can be broken down into the cost of Ellen's ticket which is **X**, and the cost of her sister's ticket, which is **.5X**. Therefore, if we can find out the value of **X** which is Ellen's ticket, we will automatically know the cost of her sister's ticket which is one-half of **X**. Hence, $**9 = X + .5X** Now for some algebraic code. Take a look first at the right side of the equation. Notice that we have **X + .5X**. Remember that we are trying to find the value of **X**, so I want **X** all by itself on one side.

One way to do this is to "factor" the right hand side of the equation like this:

$$\$9 = X + .5X$$

$$\$9 = X\,(1 + .5)$$

Now the equation looks like this:

$$\$9 = X\,(1.5)$$

Notice that we still do not have X *all by itself* on one side only, but we can accomplish this with our next move.

Divide both sides by 1.5* to get:

$$\$9/\,1.5 = X\,(1.5)/\,1.5$$

$$\$9/\,1.5 = X$$

Voile! We have X on one side only which is good because we want to know the value of X. (**Getting X on one side of the equation is an important idea because it is <u>the</u> <u>grand</u> <u>strategy</u> for solving <u>all</u> problems in algebra.**)

Now, let's turn the equation around so that X is on the left.

$$X = \$9/\,1.5$$

Rotating the equation does not change anything, because each side is equal to the other. That is the nature of an equation. So, now:

$$X = \$9/\,1.5 = \$6 \text{ (the cost of Ellen's Ticket)}$$

Therefore, if her sister's ticket is equal to one-half of Ellen's ticket, it must be worth **$3**.

*We divided both sides by 1.5 to keep the plane flying on a safe flight path with the wings level. If we divided only one side, the wings would be on a dangerous slant.

Let's try another "word problem" with Ellen and her sister

Ellen tells her sister that in her 8th grade class, there are 10 boys and 20 girls. The algebra textbook writers overheard this conversation and gathered in their office for a brainstorming session to convolute that simple fact into a "word problem" calling for algebra to solve.

Let's convolute

In Ellen's 8th grade class, there are 30 pupils. There are twice as many girls as boys. How many boys and how many girls are in Ellen's class?

Let's see how this will work into an equation, with the unknown, which is X on one side all by itself.

X = Number of Boys

2X = Number of Girls

Hence, (That's algebra talk for "Therefore.")

30 = X + 2X

Now, we want X on one side all by itself. So let's factor:

30 = X + 2X

30 = X (1 + 2)

Next, divide both sides by (1 + 2) to get:

30/ (1 + 2) = X

30/ 3 = X

10 = X

X = 10 (which is the **Number of Boys** in Ellen's class)

Since there are twice as many girls as boys, there must be **2** times **10** or **20 Girls** in Ellen's class.

Let's try another one…

Now that we know Ellen, her sister, and her classmates so well, let's try another fact about Ellen's family. Mother's Day is coming up soon and Ellen has a coupon for ten percent off a box of candy which is advertised at $20. The writers of our algebra textbook are neighbors of Ellen's family, and Ellen's little sister told them that Ellen was planning on buying the candy for their mother from money in their piggy bank. With the coupon, the cost of the box of candy will be $18.

Ahh, another opportunity to create a convoluted puzzle that will call for algebraic code to solve! Ellen and her sister have a coupon to buy a box of candy for Mother's Day.

With a coupon worth ten percent off the regular price, the girls will pay $18. What is the original cost of the candy?

Now, let's see… We need an equation with one unknown which we'll call **X**, the original cost of the candy. Let's translate into English. The discounted price of the candy is **$18**. Therefore, the original cost of the candy is **X** minus ten percent (which is .10 times **X**). Put it all together to get:

$18 = X - (.10) X

Now, factor to get:

$18 = X (1 - .10)

Rotate to get X on the left hand side and divide both sides by (1 -.10) to get:

X = $18 / .90 = $20 (the original price of the box of candy)

Counterfeit algebra

Notice that the solutions look like algebra, but the procedures are what I call "counterfeit" algebra. Here is why: We are looking for a finite answer which tells me and professional mathematicians with whom I have conferred, that the "word problems" are examples of arithmetic.

Authentic algebra deals with the infinite. Counterfeit algebra (which is really arithmetic) grabs a handful of letters from algebra and manipulates them to find a finite answer to a convoluted question that nobody will ask in their lifetime. Only members of Mensa will find these word problems fascinating puzzles to "kill" time on a rainy afternoon.

Ellen and her sister have a coin "problem"

(Not really, but our writers will surely convolute a simple fact about the piggy bank of Ellen and her sister into one of those infamous coin "problems.") Let's start with this fact: Ellen and her sister have 20 coins in their piggy bank, with twice as many nickels as dimes. How many nickels and how many dimes?

Now the moral of this story is that we have to know how many nickels and how many dimes are in the piggy bank ***before*** we try to apply some algebra. Let's try to solve this and you will see what I mean.

X is the number of dimes, and 2X is the number of nickels.

20 coins = **X** (dimes) + **2X** (nickels)

So far, it looks as if we can get a sensible answer—or can we?

Factor: **20 = X + 2X**

To get: **20 = X (1 + 2)**

20 = X (3)

20/ 3 = X

X = 20/3 = 6.66 (the number of dimes)

And since there are twice as many nickels, there must be **2** (6.66) or **13.33 nickels**. So what's wrong with this picture? The piggy bank of Ellen and her sister is supposed to have **6.66** dimes and **13.33** nickels. How can you have **6.66** dimes or **13.33** nickels? The answer does not make sense.

My point is ***we must know the answer in advance*** before we can create a convoluted puzzle that will produce numbers that make sense. In this case the number of coins must be whole numbers rather than decimals, and this puzzle will ***only*** work if the number of coins is something like 30 or 60 or 90…

30 = X (3)

30/ 3 = X

X = 30/3 = 10 (a whole number of dimes)

And therefore **20** nickels. The answer now makes sense, but since we knew the answer in advance, there's no point in asking the question!

Why convolution?

Why do the algebra textbook writers convolute? Because they cannot find authentic problems in the real lives of ordinary people that require algebraic code to solve. For years, I have invited anyone anywhere in the world to e-mail me with problems from real life that require algebra to solve. When the few responses that I did receive were submitted to professional mathematicians, they come back with, "This is an example of simple arithmetic."

Algebra is a technical code that applies to a handful of occupations: statisticians, mathematicians, engineers… and probably not physicians (except for medical researchers). Even researchers can function with success using simple arithmetic. The curious researchers want to know where the formulas come from, and that is the role of algebra.

What is algebra, really?

For professional mathematicians, algebra is a technical code for exploring properties of numbers—all or any numbers. In other words, algebra is a tool for exploring infinity, a mystical place that does not exist. Infinity is a mysterious journey that is not unlike walking towards the horizon. You can walk forever, but you will never reach the horizon. It is a journey that will never be completed. Ancient mariners were terrified of sailing too far toward the horizon for fear of falling off the end of the earth. We know now that there was nothing to worry about.

If you start counting, you can count forever and never arrive at the last number because it does not exist—or does it? Since no one has discovered the "last number," how can we be sure it does not exist? That perplexing question is one for philosophical mathematicians to ponder. You may be surprised to know that there is an esoteric specialty called the "philosophy of mathematics."

Some examples of the legitimate application of algebra

When you add two even numbers, will the result be an even number, an odd number or something else? Let's play with it:

2 + 4 = 6

4 + 6 = 10

6 + 8 = 14

Based upon our sample, If you add two even numbers, you will get an even number. Here is the mystery: Are you sure? Will you always get an even number? It seems self-evident, but we have been fooled before with number relationships that seemed to be self-evident.

For example, Pierre de Fermat, the 15th century "Prince of Amateur Mathematicians" thought that he had discovered the Silver Equation for prime numbers—one that will predict only primes, and no false primes. Here is what happened: His equation worked for primes of 3, 5, 17, 257 and 65,537, which is where he stopped his tedious paper-and-pencil computation. If a modern computer had been available to him, he would have discovered that the very next "prime" was a ***false*** prime. In mathematics, it only takes one negative case to invalidate a premise.

If you can represent a relationship such as even and odd numbers in letters, you can often test the truth for all numbers, which means we can actually explore infinity, which is another name for eternity. To illustrate, consider this neat little "proof." First, we can represent even numbers with letters such as 2n. How does this work? Well, n can represent any natural counting numbers such as 1, 2 or 3, etc. If you multiply any whole number by 2 the result will always be an even number. Try it and see. I acknowledge that this is a premise, a starting assumption. The only "proof" I can offer is that no one has ever yet found any whole number multiplied by 2 that is not an even number.

The point is that we can represent ***all*** even numbers by 2n. Because I said "all" in the previous sentence, that means that we're now operating in that mystical territory of "infinity." Let's add two even numbers represented by infinity and see what happens:

$$2n + 2n = 4n$$

Here we add an even number and an even number to get 4n. But I do not recognize 4n. It does not look like an even number which is 2n and it does not look like an odd number which is 2n + 1.

Now factor to get: $2(n + n) = 4n$

Divide both sides by **2** to get:

$$2n = 2n$$

We wind up with an even number. That was successful!

Let's play with it some more. What if we multiply three even numbers. Will the result be an even number, an odd number or something else? I don't know what will happen myself. So let's try it...

$$2n + 2n + 2n = 6n$$

Now I don't recognize 6n as a number that is even, odd or something else because it looks different from 2n which is an even number.

Now factor to get: $2(n + n + n) = 6n$

Which reduces to: $2(3n) = 6n$

Divide both sides by **3n** to get: $2 = 2$ Again, we get an even number.

Well then, what do you recommend for our children?

I know of three options we can try. The first is to make algebra an elective rather than a mandatory course for all students. Many students enjoy the intricate pattern-making activity of algebra. These students find the patterns fascinating apart from any synthetic attempt to make the product relevant. They will enjoy the course.

Algebra should be declassified from its current status as "something everyone has to know" to "here is another interesting elective you may enjoy along with art, botany, or sports." But, what about those who "need to know" for work in the physical sciences as chemistry and physics? The key words here are "need to know." Our model should be the police academy where my son graduated after earning a degree from San Jose State University. Police officers "need to know" a huge chunk of law to be effective in their work. As candidates progress through the police academy, they internalize statute after statute on a "need to know" basis. I see a similar strategy in chemistry or physics.

As we move through the course, when we "need to know," the mind opens up a huge window. We seem to understand information immediately in almost one exposure.

But, don't we want our children to be math-literate?

Of course we do. But how are we going to do this? Obviously, our current attempt at "forcing" the information into young learners is not working. Then consider this: We have successful electives that attract thousands of students. The names of these courses: Art Appreciation and Music Appreciation. It is time for a new elective called Mathematics Appreciation.

In my book, The Super School, I suggest that the content of this new elective should be the dramatic stories of scientists and mathematicians. For example, there is intrigue in the story of Bertrand Russell and Alfred North Whitehead who wrote a prize-winning volume to explain why 1 + 1 = 2. How can someone write an entire book on something as obvious as 1 + 1 = 2?

Then there is René Descartes, the 15th century French soldier and mathematician, who discovered the "Atlantis" of the mathematical world. For centuries, mathematicians believed there was no connection between geometry and algebra. Descartes felt that his colleagues were wrong. He began to search for the mysterious connection that he believed was there, but invisible. In his diary, Descartes wrote, " One night when I was in a deep sleep, the Angel of Truth came to me and whispered the secret connection between geometry and algebra." Without this revelation, our world as we know it, would disappear. There would be no architecture, engineering or science. All of our technological, scientific, and medical marvels were discovered because of a visit from Descartes' Angel of Truth.

Carl Friedrich Gauss, recognized as the Prince of Mathematics, wrote his thoughts in a scientific diary that is now revered as "the most precious document in all mathematics." One of his famous discoveries was to see a hidden pattern in numbers that was invisible to mathematicians for hundreds of years. We must include in our stories the Michelangelo of science and mathematics, Sir Isaac Newton. He discovered calculus, the composition of white light, and the laws of gravity. Sir Isaac believed that God must make some personal adjustments from time to time to keep planets in their orbit.

Famous scientists and mathematicians can inspire young people

Laura Nickel and Curt Noll were only 15 years old when they heard the story of the Chinese mathematician Chen Jin-Run. This person, who is still living, dedicated his professional life to exploring the fundamental theorem of arithmetic that involves prime numbers. All numbers seem to be composed of certain other numbers called primes. What fascinated Nickel and Noll was the notion that primes are a sort of DNA of all numbers. The two high school students were surprised that no pattern had yet been found to predict the highest prime.

They set out to find a number higher than the highest known prime recorded in the Guinness Book of Records—a number that contained 6,002 digits—calculated by Dr. Bryant Tuckerman of IBM. Mathematics professors warned them that their project might be doomed to failure, but they were determined to prove the experts wrong. After 2,000 hours of work and 44 computer tests, they found the elusive number which was confirmed by theoretical mathematicians at the University of California's Berkeley campus. Dr. Tuckerman telephoned them with his congratulations. If a student is to be wildly passionate about mathematics, the student must have the opportunity to experience the romance of mathematics. Romance comes first. Later comes the skills.

This article was excerpted from James J. Asher's books:
Brainswitching: ***Learning on the right side of the brain,***
The Super School,
A Simplified Guide to Statistics for Non-Mathematicians, and
The Weird and Wonderful World of Mathematical Mysteries

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Published by:
Sky Oaks Productions, Inc.
P.O. Box 1102
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Fax: (408) 395-8440
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